01 Aug 1992

CCFSS Technical Bulletin August 1992

Wei-Wen Yu Center for Cold-Formed Steel Structures

CCFSS Technical Bulletin

Vol. 1, No. 2 August 1992

Four Span Continuous Z-Purlin Design Example

The following design example was prepared by Subcommittee 26 on Design Manual of the AISI Committee on Specifications. It is intended to illustrate the design of continuous Z-purlins subjected to downward and upward loads.

Given:

1. Four span Z-purlin system using laps at interior support points to create continuity (see Figures 1 and 2).
2. Exterior spans, t = 0.098 inches
   Interior spans, t = 0.084 inches
3. Roof covering is attached with through fasteners along entire length of purlins.
4. F_y = 55 ksi

Required:

1. Check the design against AISI Specification criteria for downward loads.
2. Check the design against AISI Specification criteria for loads acting upward.

Solution:

1. Assumptions for Analysis and Application of Specification Provisions

The AISI Specification does not define the methods of analysis to be used; these judgements are the responsibility of the designer. The following assumptions are considered good practice but are not intended to prohibit other approaches:

a. The purlins are connected within the lapped portions in a manner that achieves full continuity between the individual purlin members.

b. The continuous beam analysis to establish the shear and moment diagrams assumes continuous non-prismatic members in which the I_x values within the lapped portions is the sum of the individual members.

c. The strength within the lapped portions is assumed to be the sum of the strengths of the individual sections.

d. The attachment of the roof covering to the purlin provides continuous lateral support to the top flange.

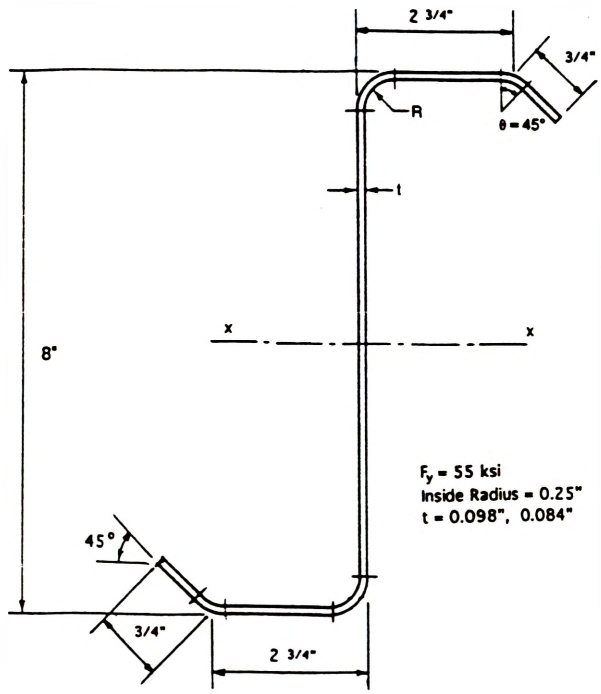
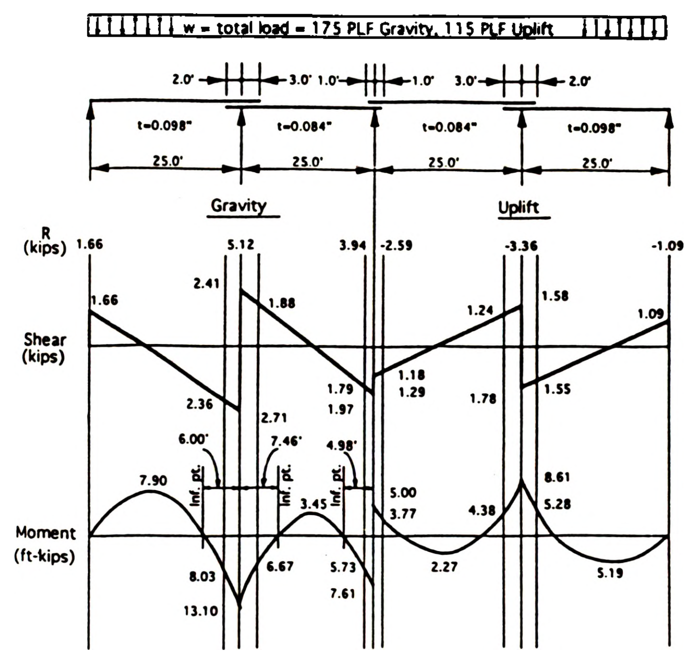


Figure 1



Note: Lap dimensions are shown to connection points of the purlin.
The total load includes allowance for purlin weight.

Figure 2

e. For gravity loads, the compression (bottom) flange at and near the interior supports is assumed to be fully braced between the support and the end of the lap. The inflection point is also assumed to be a braced point for the bottom flange, therefore C_b = 1.75.

2. Calculation of Section Properties:

Based on the design procedures illustrated in Examples No. 4 and No. 4A of the AISI Cold-Formed Steel Design Manual, the following sectional properties can be obtained for the two Z-sections:

For $t = 0.084$ in.
$I_x = 11.9$ in.^{4}
$S_f = 2.97$ in.^{3}
$S_e = 2.55$ in.^{3}
$I_y = 2.15$ in.^{4}

For $t = 0.098$ in.
$I_x = 13.7$ in.^{4}
$S_f = 3.44$ in.^{3}
$S_e = 3.16$ in.^{3}
$I_y = 2.48$ in.^{4}

A. Check the Design Against AISI Specification Criteria for Downward Loads

1. Allowable Moments Based on Initiation of Yielding (Section C31.1.1(a))

For t = 0.084 in.

$$M_a = \frac{S_e F_y}{1.67} = 84.0 \text{ in.-k or } 7.00 \text{ ft-k}$$

For t = 0.098 in.

$$M_a = \frac{S_e F_y}{1.67} = 104.1 \text{ in.-k or } 8.68 \text{ ft-k.}$$

2. Lateral Buckling Strength for Bending Only (Section C3.1.2)

End Span:
Maximum positive moment = 7.90 ft-k
< 8.68 ft-k OK

Negative moment at support = 13.1 ft-k
< (8.68 + 7.00) = 15.68 ft-k OK
Negative moment at lap = 8.03 ft-k
Determine the allowable moment using distance from inflection point to lap as the unbraced length (Section C3.1.2(b)).

L = 6.00 - 2.00 = 4.00 ft = 48 in.,

$$I_{yc} = \frac{I_y}{2} = \frac{2.48}{2} = 1.24 \text{ in}^4$$

$$C_b = 1.75$$

$$M_e = \frac{\pi^2 E C_b d I_{yc}}{2L^2}$$

$$= \frac{\pi^2 \times 29500 \times 1.75 \times 8 \times 1.24}{2 \times 48^2}$$

$$= 1{,}097 \text{ in.-k}$$

$$M_y = S_f F_y = 3.44 \times 55 = 189.2 \text{ in.-k}$$

Since $M_e = 1{,}097$ in.-k > 2.78 M_y = 525 in.-k,

$$M_c = M_y = 189.2 \text{ in.-k}$$

$$M_n = S_c \frac{M_c}{S_f} = 3.16 \times \frac{189.2}{3.44} = 173.8 \text{ in.-k}$$

$$M_a = \frac{173.8}{1.67} = 104.1 \text{ in.-k or } 8.68 \text{ ft-k}$$

8.03 ft-k < 8.68 ft-k OK

Interior span:
Maximum positive moment
= 3.45 ft-k < 7.00 ft-k OK
Negative moment at center support
= 7.61 ft-k < (7.00 + 7.00) = 14.00 ft-k OK
Negative moment at end of lap = 6.67 ft-k
Determine the allowable moment using distance from inflection point to lap as the unbraced length (Section C3.1.2(b)).

L = 7.46 - 3.00 = 4.46 ft or 53.5 in.,

$$I_{yc} = \frac{I_y}{2} = \frac{2.15}{2} = 1.08 \text{ in.}^4$$

$$M_e = \frac{\pi^2 \times 29500 \times 1.75 \times 8 \times 1.08}{2 \times 53.5^2}$$

$$= 765 \text{ in.-k}$$

$$M_y = 2.97 \times 55 = 163.4 \text{ in.-k}$$

Since M_e = 765 in.-k > M_y = 163.4 in.-k,

$$M_c = M_y = 163.4 \text{ in.-k}$$

$$M_n = 2.55 \times \frac{163.4}{2.97} = 140.3 \text{ in.-k}$$

$$M_a = \frac{140.3}{1.67} = 84.0 \text{ in.-k or } 7.00 \text{ ft-k}$$

6.67 ft-k < 7.00 ft-k OK

3. Strength for Shear Only (Section C3.2)

For t = 0.084 in. and h = 7.33 in., h/t = 87.3

$$\frac{h}{t} > 1.38 \times \sqrt{\frac{29500 \times 5.34}{55}} = 73.9$$

$$V_a = \frac{0.53\ EK_v\ t^3}{h}$$
$$= \frac{0.53 \times 29500 \times 5.34 \times 0.084^3}{7.33}$$
$$= 6.75\ k$$

For t = 0.098 in. and h = 7.30 in., h/t = 74.5

$$V_a = \frac{0.53 \times 29500 \times 5.34 \times 0.098^3}{7.30}$$
$$= 10.8\ k$$

End Span:
Shear at start of lap = 2.36 k
< 10.8 k OK
Shear at interior support = 2.71 k
< (6.75 + 10.8) = 17.55 k OK
Interior Span:
Shear at end of lap = 1.88 k < 6.75 k OK
Shear at center support = 1.97 k
< (6.75 + 6.75) = 13.50 k OK

4. Strength for Combined Bending and Shear (Section C3.3)

$$\left(\frac{M}{M_{axo}}\right)^2 + \left(\frac{V}{V_a}\right)^2 \le 1.0$$

End Span:
At start of lap, $\left(\frac{8.03}{8.68}\right)^2 + \left(\frac{2.36}{10.8}\right)^2$

= 0.90 < 1.0 OK

At interior support,

$$\left(\frac{13.10}{8.68 + 7.00}\right)^2 + \left(\frac{2.71}{10.8 + 6.75}\right)^2$$

= 0.72 < 1.0 OK

Interior Span:
At end of lap, $\left(\frac{6.67}{7.00}\right)^2 + \left(\frac{1.88}{6.75}\right)^2$

= 0.99 < 1.0 OK

5. Web Crippling Strength (Section C3.4)

The following assumes a bearing length of 5 inches.

At interior support use Eq. C3.4-4 of the AISI Specification

For t = 0.084 in., N/t = 5/0.084 = 59.5

$$P_a = t^2 k C_1 C_2 C_\theta \left[291 - 0.40\frac{h}{t}\right]\left[1 + 0.007\frac{N}{t}\right]$$
$$= 0.084^2 \times \frac{55}{33}\left(1.22 - 0.22 \times \frac{55}{33}\right)$$
$$\times \left(1.06 - 0.06 \times \frac{0.25}{0.084}\right)$$
$$\times 1.0\left[291 - 0.40 \times \frac{7.33}{0.084}\right]$$
$$\times [1 + 0.007 \times 59.5]$$

P_a = 3.21 k

For t = 0.098 in., N/t = 5/0.098 = 51.0

$$P_a = 0.098^2 \times \frac{55}{33}\left(1.22 - 0.22 \times \frac{55}{33}\right)$$
$$\times \left(1.06 - 0.06 \times \frac{0.25}{0.098}\right)$$
$$\times 1.0\left[291 - 0.40 \times \frac{7.30}{0.098}\right]$$
$$\times [1 + 0.007 \times 51.0]$$

P_a = 4.39 k

Reaction = 5.12 k < (3.21 + 4.39) = 7.60 k OK

6. Combined Bending and Web Crippling (Section C3.5)

$$1.2\left(\frac{P}{P_a}\right) + \left(\frac{M}{M_{axo}}\right) \le 1.5$$

$$1.2\left(\frac{5.12}{3.21+4.39}\right) + \left(\frac{13.10}{8.68+7.00}\right)$$
$$= 1.64 > 1.5\ \text{NG}$$

This indicates the design is NOT SATISFACTORY for the given gravity loads. A bearing stiffener could be added to correct this deficiency or a different section selected.

B. Check the Design Against AISI Specification Criteria for Uplift Loads

1. Strength for Bending Only (Section C3.1.3)

$M_n = R\ S_e\ F_y$

$R = 0.70$

For t = 0.084 in.

$M_n = 0.70 \times 2.55 \times 55 = 98.2$ in.-k or 8.13 ft-k

$M_a = M_n / 1.67 = 8.13 / 1.67 = 4.87$ ft-k

For t = 0.098 in.

$M_n = 0.70 \times 3.16 \times 55 = 121.7$ in.-k or 10.1 ft-k

$M_a = M_n / 1.67 = 10.1 / 1.67 = 6.05$ ft-k

For wind load conditions, the forces may be multiplied by 0.75 for strength determination (Section A4.4). Thus the calculated moments can be reduced.

End Span:
Moment near center of span = 5.19 x 0.75 = 3.89 ft-k < 6.05 ft-k OK

Interior Span:
Moment near center of span = 2.27 x 0.75 = 1.70 ft-k < 4.87 ft-k OK

2. Other Comments

Based on the previous checks for gravity loads, it may be seen that the wind load satisfies other appropriate Specification criteria, therefore this design is SATISFACTORY for the uplift loads.

Center for Cold-Formed Steel Structures
University of Missouri-Rolla
Rolla, MO 65401-0249 USA

UMR
Missouri's Technological University